USPT Problem 4

String Bean Theorists
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Problem 4

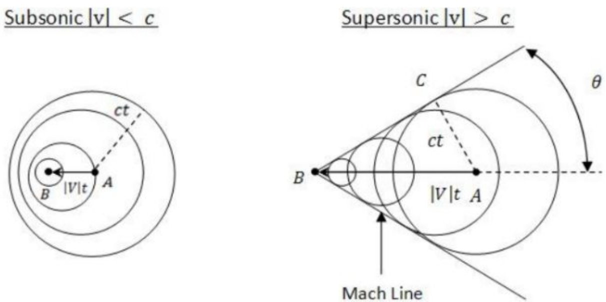
Hydraulic jump white hole is when a steady stream of water from a tap hits the sink, the water spreads in a circular disc bounded by a region where the water height is greater than its surroundings, as seen in the Figure. This so-called hydraulic jump is analogous to a white hole, the time-reversed version of a black hole, in the sense that surface waves cannot enter the disc against the flow, whilst there is a natural outward flow. **Explain the physics behind the hydraulic jump, and how its properties can be matched to those of a white hole. Perform an experiment in order to verify the correspondence. Is it possible to make a hydraulic jump with liquids other than water? When does the white hole analogy break down?**

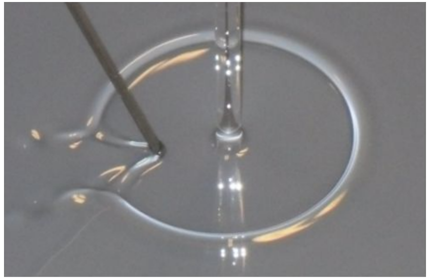
Theory for hydraulic jump - why the jump occur

- The jump occurs because the fast flow of water rapidly slows down as it spreads, and it starts to pile up on top of itself
- So the height increases and jump occurs

Theory for hydraulic jump - Supercritical and Subcritical Area

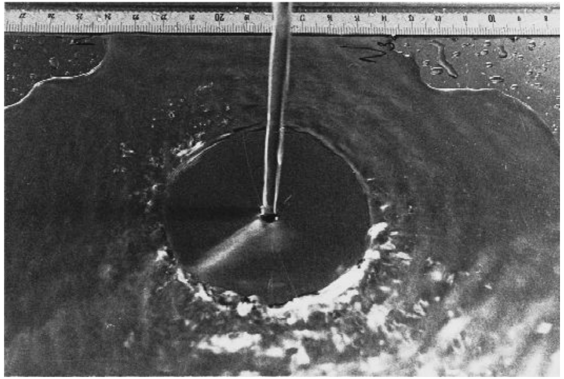
- Supercritical area - circular hydraulic jump
- Subcritical area - outer area with high water layer height
- Deceleration as water travel across supercritical area and across subcritical area
- Supercritical: radial fluid velocity > propagation speed of surface wave
    - For this reason, ripples only propagate downstream
- Subcritical: radial fluid velocity < propagation speed of surface wave
    - Ripples can propagate in all directions
- Analogy to shockwave
    - Triangular shape
    - Transition from supercritical to subcritical
- The transition characterized as a jump





Analogy to White Hole

- Previous physics properties prove that circular hydraulic jump constitute a two-dimensional hydrodynamic white hole
- Any surface waves (ripples) in the subcritical area are trapped outside the circular jump
- Similar to how black hole would trap light, white hole will prevent light from entering the horizon.
- Water only flows outward from the center - white hole will only emit mass from inside to outside and emitting away from the center.



Analogy Theory - Mass and Velocity

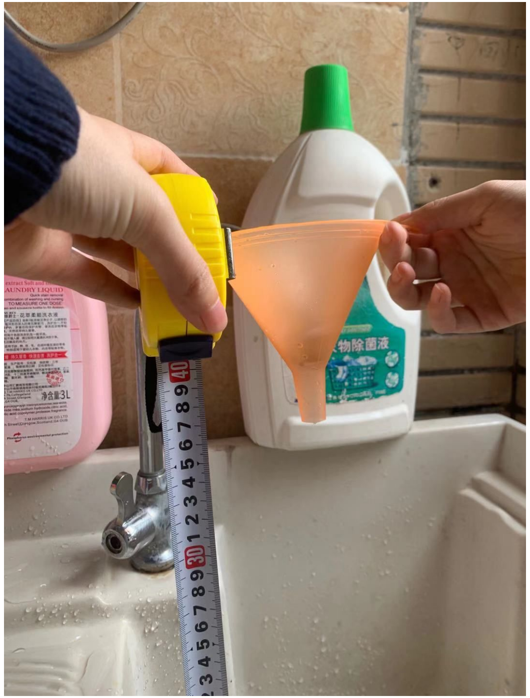
- White hole is a time-reversed version of black hole
- Black hole property:
  - Increase in mass would increase the radius of event horizon of black hole
  - Mass that are falling into the black hole are experiencing a higher velocity
- Matching a white hole:
  - Increase in mass would increase the radius of event horizon of white hole
    - Increase in volume flow rate increases the radius of hydraulic jump
  - Mass that are emitted are experiencing a higher velocity
    - Increase in radial fluid velocity will increase the radius of hydraulic jump

Analogy Theory - Rotation

- Property for black hole:
  - spinning ones are with a smaller event horizon - spinning white hole will have a smaller event horizon
- Incorporate spinning in hydraulic jump
  - When the surface the water strikes on is rotating, what will happen to the radius of hydraulic jump?

Volume flow rate vs. Radius experimental setup

- Funnel with different radius (length of the narrow bottom varies by 2cm)
- Measuring the height and let bottom of the funnel be on the same level, so keeping d the same
- Keep the tap on by the same amount, so water velocity from the tap will be the same

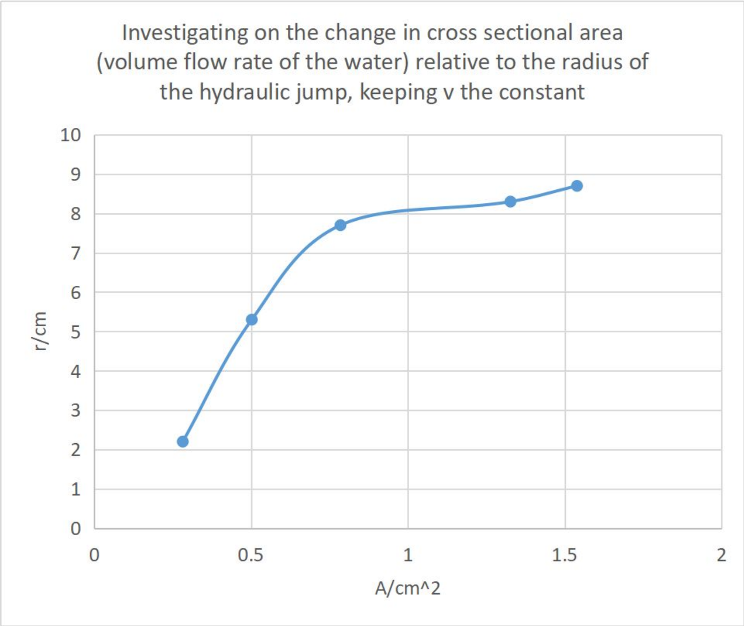


Volume flow rate (q)

- q=Av where A is cross sectional area, v is flow velocity
- Keeping v the constant by keeping the flow from the tap constant
- A is dependent variable

Volume flow rate vs. Radius experimental data

A/cm^2	r/cm
0.2827	2.2
0.5027	5.3
0.7854	7.7
1.3273	8.3
1.5394	8.7

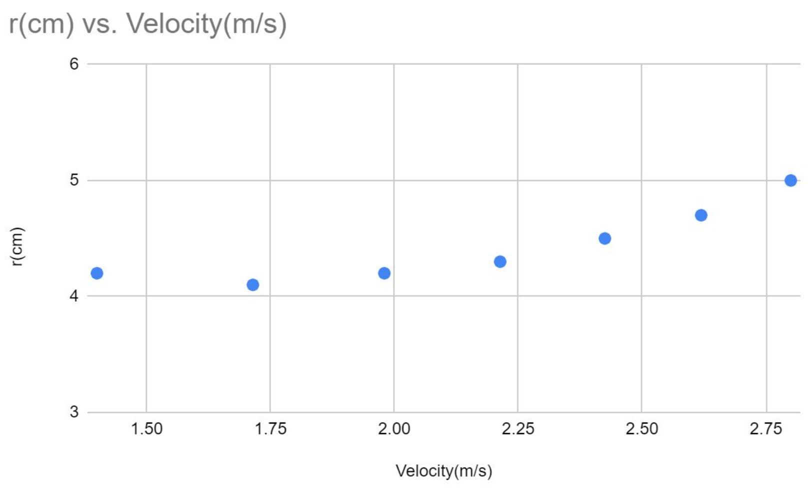


Velocity vs. Radius Experimental Setup

- Regular home sink
- Multiple rulers
- Small stream of water
  - The initial velocity of water stream is negligible as shown on the right picture, there are almost no horizontal velocity as water is coming out

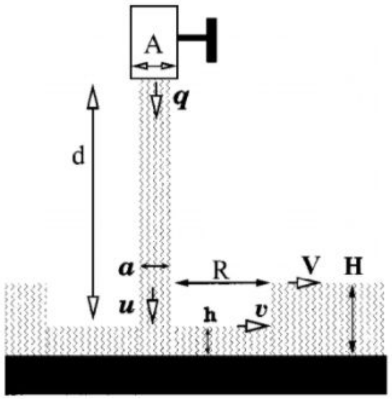
Velocity vs. Radius Experiment Data

Height(cm)	Velocity(m/s)	r(cm)
10	1.40	4.2
15	1.71	4.1
20	1.98	4.2
25	2.21	4.3
30	2.42	4.5
35	2.62	4.7
40	2.80	5



Equation from Research Paper

- As d increase, R increase
- As q increase, R increase



$$R = \frac{4q^2\sqrt{\frac{\pi^2 g d}{8q^2} + \frac{1}{A^4}}}{\pi^2 g H^2}.$$

Bréchet, Yves & Néda, Zoltán. (1999). On the circular hydraulic jump. American Journal of Physics. 67. 723-731. 10.1119/1.19360.

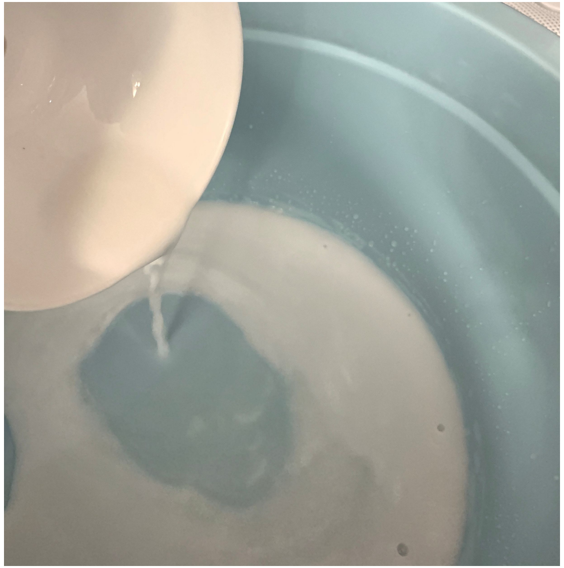
Future Experiment Consideration

- Black hole and white hole all have self-rotation
- Possible experiment is to have a fast rotating flat plate
  - Mimic rotating white hole
  - Observe whether rotation has an effect, increasing or decreasing the radius of hydraulic jump
- Home-made experiment failed
  - Used cake making stand, but angular velocity was too small to observe a change in the radius of hydraulic jump
  - Also it was not constant angular velocity, since it was rotated by hand, and will be slowed by friction

Hydraulic jump with other liquids - oil

Hydraulic jump with other liquids - non-Newtonian fluid

- Corn starch

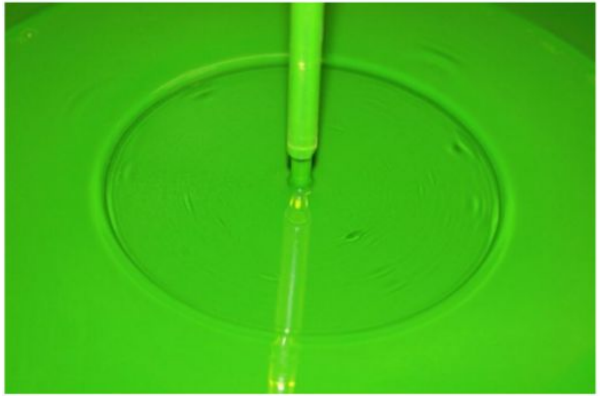


Analogy Breakdown - Viscosity

- When viscosity increases, hydraulic jump starts to break down
  - For hydraulic jump to occur, inner region: radial fluid velocity > propagation speed of surface wave
  - Viscosity is large, surface velocity will be small
    - Radial fluid velocity will not be bigger than propagation speed of surface wave
    - For hydraulic jump to occur, there needs to be a transition from **radial fluid velocity > propagation speed of surface wave** to **radial fluid velocity < propagation speed of surface wave**
- The hydraulic jump no longer exist - the analogy breakdown

Analogy Breakdown - Concentrating Stream

When decreasing the water-jet radius (increased in momentum), hydraulic jump may become unstable, thus breaking the circular white hole pattern



Citation

Bréchet, Yves & Néda, Zoltán. (1999). On the circular hydraulic jump. American Journal of Physics. 67. 723-731. 10.1119/1.19360.

Ahmad Saberi, Mohammad Reza Mahpeykar, A.R. Teymourtash, Experimental measurement of radius of circular hydraulic jumps: Effect of radius of convex target plate, Flow Measurement and Instrumentation, Volume 65, 2019, Pages 274-279, ISSN 0955-5986, https://doi.org/10.1016/j.flowmeasinst.2019.01.011.

arXiv:1203.6505v1 [gr-qc] 29 Mar 2012

Some photo from Google